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Impulse Noise Removal using Sparse Representation with Fuzzy Weights

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***Abstract*—Many impulse noise removal algorithms do not reach good denoising performance mainly due to the imperfect filters they adopted. In this paper, the popular used sparse representation model is extended for impulse noise removal by using a fuzzy weight matrix. This fuzzy weight is used to describe the noise-like level of the current pixel, and to determine how much information of this pixel should be used in the sparse land model. Besides, a regularization term which counts the proximity between the reconstructed image and the noisy image is also added into the sparse model. This makes the proposed model more robust to the noise detector which generates the fuzzy weight matrix. Moreover, unlike other sparse model, the dictionary used in our model is trained from some reference images that keep the similar structure information of the original image. Therefore, it is more suitable for reconstructing the original image. Simulation results show that our method is superior to all the tested state-of-the-art impulse noise removal methods.**

***Index Terms*—image denoising, impulse noise, sparse representation, weighted sparse-land model.**

I. Introduction

Impulse noise is universally introduced into images due to the acquisition processing, timing errors in analog-to-digital conversion, and communication in noisy channels. It is very important to remove noise in images because the noisy images cannot directly used for the subsequent image processing such as image compression, segmentation, edge detection, and recognition.

For impulse noise removal, median (MED) [1] filter, famous for its simplicity and high computation efficiency, is one of the most common used methods. The shortcoming of MED is that it always remove image details due to its replacement of each pixel with the median value. Therefore, for better performance, some extensions of median filter [2], [3] have been designed. These filters assign large weights for favorite pixels to preserving the desired image details. Unfortunately, they still degrade the image quality because they process every pixel in the noisy image without considering whether the current pixel is impulse noise or not.

Later, due to the characteristic of impulse noise is that it changes only a portion of pixel values but leave the rest of pixels remain the same, many denoising methods allied with noise detectors emerged as required, examples including the conditional signal-adaptive median (CSAM) filter [4]. Luo-iterative method [5], directional weighted median (DWM) filter [6], contrast enhancement-based filter (CEF) [7], histogram based method [8], switching bilateral filter (SBF) [9]. Using noise detectors, these techniques first judge whether a selected pixel is impulse noise, then noise pixels are replaced by their estimations and clean pixels are unchanged. Thanks to noise detectors, these techniques can preserve more image details. However, they still degrade the image quality due to the imperfect filtering tools they used.

Recent years, sparse representation technique has been successfully used for removing Gaussian noise [10], [11]. The basic idea is that natual images can be sparsely decomposed by a redundant basis called dictionary, hence the noisy images can be reconstructed from the sparse coefficient. It has been also shown that the image inpainting problem can be perfectly solved by sparse representation [12]. This accelerates the development of the technologies of impulse noise removal using sparse land model [13], [14]. These methods first detect the impulse noise in the noisy image and the detected noisy pixels are treated as holes; then the noisy image is inpainted by the sparse representation model. Though they show good performance in removing most of impulse noise, these inpainting based methods restricted by two shortcomings. First, the pixel is just judged by two states, noisy or clean. However, it is more suitable to describe a pixel by a fuzzy rule since the inherent feature of impulse noise is uncertainty [15], that is, describe it as noise with a fuzzy weight. Second, the pixels detected as noise are not used to reconstruct the restored image. However, for a noise detector, it is very easy to wrongly detect an information pixel to be noise, hence these pixels still should be used in the sparse land model.

In this paper, we propose a sparse representation model with fuzzy weight to remove impulse noise in images. The fuzzy weight is assigned for each pixel to describe the level of noise. With this weight matrix, the noisy image is integrated into the sparse model to reconstruct the original image. Besides, dictionary chosen is also a crucial problem for sparse representation. In this paper, we propose a framework of training the dictionary from some reference images that keep the similar textures and structures with the original image. Therefore, the trained dictionary is more suitable to represent the noisy image.

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The rest of this paper is organized as following, the proposed method is described in details in Section II. Experimental results are shown in Section III. Finally, the conclusion is presented in Section IV.

II. The Proposed Algorithm

A. Impulse Noise Model

The characteristic of impulse noise is that, for an image corrupted by impulse noise, just a portion of pixel values are changed but the remaining are unchanged. Let $y_{i,j}$ and $x_{i,j}$ be the (i,j)-th pixel values in the clean and noisy images, and $[m_{min}, m_{max}]$ be the dynamic range of pixel values, then the impulse noise can be modeled as,

$$x_{i,j} = \begin{cases} y_{i,j} & p = 1 - p_{noise} \\ n_{i,j} & p = p_{noise} \end{cases} \tag{1}$$

where $n_{i,j}$ is the noise value which has no relationship with $y_{i,j}$, p_{noise} is the noise probability. Generally speaking, there are two kinds of impulse noise, one is fixed-valued impulse noise, also named as salt & pepper noise; the other is random-valued impulse noise. For salt & pepper noise, $n_{i,j}$ equals to n_{min} or n_{max}, while for random-valued impulse noise, $n_{min} \leq n_{i,j} \leq n_{max}$. In 8-bit grayscale images, $n_{min} = 0$ and $n_{max} = 255$.

B. Weighted Sparse Representation Model

Recently, the sparse representation emerged as a powerful mathematical model has been widely used in image processing, such as image denoising [10], super-resolution [16], recognition [17] and so on.

For image denoising, sparse representation tries to reconstruct a noisy image by a weighted combination of several images bases. These image bases come from a pre-defined database which is called dictionary.

For Gaussian noise removal, the noisy image is first divided into small overlapping patches, then each patch is processed by the following energy minimization model:

$$\hat{\alpha}_{i,j} = \arg\min_{\alpha} \sum_{i,j} \| D\alpha_{i,j} - R_{i,j}X \|_2^2 + \sum_{i,j} \lambda \| \alpha_{i,j} \|_0 \tag{2}$$

where X, the noisy image with size of $\sqrt{N} \times \sqrt{N}$, $D \in R^{n \times K}$ ($n < K$), is the redundant dictionary, $R_{i,j}$ denotes a $n \times N$ matrix which is used to extract the (i,j)th patch from the image, λ is a positive constant which is used to balances the sparsity of the coefficient and the fidelity term, $\| \cdot \|_0$ is the l_0 norm which counts the number of non-zero coefficients in α, and α is the representation coefficient which is expected to be sparsity.

Solving (2) is known to be a NP-hard problem, because it is a non-convex function. Therefore, in practice, to change the non-convex optimization problem into a convex optimization problem, the l_0 norm is usually replaced by the l_1 norm, as followed,

$$\hat{\alpha}_{i,j} = \arg\min_{\alpha} \sum_{i,j} \| D\alpha_{i,j} - R_{i,j}X \|_2^2 + \sum_{i,j} \lambda \| \alpha_{i,j} \|_1 \tag{3}$$

When the coefficient $\hat{\alpha}_{i,j}$ is solved, the reconstructed patch $\hat{x}_{i,j} = D\hat{\alpha}_{i,j}$ is treated as the denoised patch and replaces the noisy patch $y_{i,j}$ in the (i,j)th position. Spontaneously, the reconstructed image is then obtained by averaging these denoised patches in each position.

The denoised results will be better if the global prior is added into (2), therefore, a generalization model of (2) is designed as,

$$(\hat{Y}, \hat{\alpha}) = \arg\min \frac{\lambda_1}{2} \| Y - X \|_2^2 + \sum_{i,j} \frac{1}{2} \| D\alpha_{i,j} - R_{i,j}Y \|_2^2 + \sum_{i,j} \lambda_2 \| \alpha \|_1 \tag{4}$$

In the above model, Y is denoised image (estimation of X). The first term is the global energy function whose minimization demands the proximity between the reconstructed image and the noisy image.

Although the above model has very good performance in suppress Gaussian noise, it failed seriously in removing impulse noise. The reason is that this model minimizes the energy function of each pixel in image without considering if it is noise or not. Because these pixels corrupted by impulse noise have no relationship with the original pixels and their use in the energy minimization function can only be harmful, these corrupted pixels should be excluded from the energy function. Motivated by this, in this section, we propose a sparse model with fuzzy weights for impulse noise removal, as follows.

$$\begin{aligned}(\hat{Y}, \hat{\alpha}) = \arg\min_{Y,\alpha} & \frac{\lambda_1}{2} \| W \otimes (Y - X) \|_2^2 \\ & + \frac{\lambda_2}{2} \| (I - W) \otimes (Y - X_{med}) \|_2^2 \\ & + \sum_{i,j} \frac{1}{2} \| D\alpha_{i,j} - R_{i,j}Y \|_2^2 + \sum_{i,j} \lambda_3 \| \alpha \|_1 \end{aligned} \tag{5}$$

In this model, W is a weight matrix generated by a noise detector, and with the same size of X, whose element $0 \leq w_{i,j} \leq 1$ denotes how the level fo noise of the current pixel $y_{i,j}$ is. $w_{i,j} = 0$ means the pixel $y_{i,j}$ is detected as absolutely impulse noise; $w_{i,j} = 1$ indicates (i,j)th pixel $y_{i,j}$ is identified as clean pixel. And $0 < w_{i,j} < 1$ denotes $y_{i,j}$ with $w_{i,j}$ probability to be an impulse noise. X_{med} is the median output of X. The symbol $\otimes$ denotes the element-wise multiplication.

In the following discussion, we assume the redundancy dictionary D is known, actually, in our method, the dictionary is trained from some image database that relates to the original image.

The model in (5) can be solved by an iteratively relaxation method, which updates the representation coefficient α and the estimation image Y individually by relaxing one component while fixing the other untouched. 1) Fix Y, α is updated by,

$$\alpha = \arg\min_{\alpha} \frac{1}{2} \sum_{i,j} \| D\alpha_{i,j} - R_{i,j}Y \|_2^2 + \sum_{i,j} \lambda_3 \| \alpha \|_1 \tag{6}$$

this is a convex function known as spare coding, and it can be solved by an iterative shrinkage algorithm which updates the coefficient by a soft shrinkage operator,

$$\alpha_{i,j}^{(k+1)} = S_{\lambda_2}\left(\frac{1}{c}D^T\left(R_{i,j}Y - D\alpha_{i,j}^{(k)}\right) + \alpha_{i,j}^{(k)}\right) \quad (7)$$

where the parameter c is a constant introduced by the surrogate functions, and S is the soft threshold operator, defined by,

$$S_\lambda(x) = \begin{cases} x-\lambda; & x>\lambda \\ 0; & -\lambda \le x \le \lambda \\ x+\lambda; & x<-\lambda \end{cases} \quad (8)$$

2) Fix α, the update of Y becomes the following optimization problem.

$$\hat{Y} = \arg\min \frac{\lambda_1}{2}\|W \otimes (Y-X)\|_2^2 + \frac{\lambda_2}{2}\|(I-W)\otimes(Y-X_{med})\|_2^2 + \frac{1}{2}\sum_{i,j}\|D\alpha_{i,j} - R_{i,j}Y\|_2^2 \quad (9)$$

the above subproblem is a quadratic optimization problem which is convex and differentiable. Hence, the solution can be obtained by calculating the deviation with respect to Y and setting it to zero.

Finally, the impulse noise removal algorithm is summarized in algorithm 1.

Algorithm 1. The proposed impulse noise removal algorithm

Input: The noisy image X; Maximum number of iterations L_{max}; Initialize $l=1$, $Y^{(l)}=X$

Iterative on l ($l = 1, 2, \cdots L_{max}-1$);

1: Update coefficient: compute $\alpha^{(l+1)}$ by the iteration shrinkage in (7), as

$$\alpha_{i,j}^{(l),(k+1)} = S_{\lambda_2}\left(\frac{1}{c}D^T\left(R_{i,j}Y^{(l)} - D\alpha_{i,j}^{(l),(k)}\right) + \alpha_{i,j}^{(l),(k)}\right)$$

2: Update the denoised image: calculate $Y^{(l+1)}$ by Eq. (9).

Output: The restored image $\hat{Y} = Y^{L_{max}}$

Note that the proposed weighted sparse model can be integrated with any noise detector to remove the impulse noise. In this paper, we choose the detector developed in the reference [6] to generate the weight W.

C. Dictionary Design

Until now, we assume that the dictionary is known. Actually, dictionary chosen is an intellectual problem, which concerns the performance of the sparse-land model [12].

In practical application, the dictionary D can be obtained by two ways. One is chosen from the pre-defined transforms such as DCT, wavelet, contourlet, and curvelet and so on. The other is trained from some database or the noisy image itself. It has been proved that the trained dictionaries are better than the pre-defined transform dictionaries, and the dictionaries trained from noisy image itself are better than those from database. Unfortunately, the image corrupted by impulse noise is no longer suitable for training dictionary because of the independent of impulse noise. Therefore, in this paper, we training the dictionary from some reference images. These reference images are the output of some impulse noise removal methods, hence, they have less noise but the similar texture and detail information with the original image. Therefore, they are more suitable for representing the noisy image.

TABLE I: Comparison of restoration results in PSNR for images corrupted by random-valued impulse noise

Method		Luo's [5]	CEF [7]	DWM [6]	SDOOD [21]	SBF [9]	Proposed
lena	30%	33.75	34.05	34.62	30.61	31.82	**34.64**
	40%	30.77	32.11	32.34	29.60	30.04	**33.19**
	50%	27.16	29.76	29.32	27.89	27.18	**29.63**
barbara	30%	24.62	23.67	24.20	23.86	23.75	**26.35**
	40%	23.21	23.22	23.62	23.11	23.46	**24.73**
	50%	21.90	22.93	23.05	22.80	22.92	**23.70**
house	30%	36.40	37.60	37.81	35.31	36.27	**39.97**
	40%	33.43	35.23	36.27	33.94	33.67	**38.08**
	50%	29.72	33.36	33.24	31.22	29.97	**34.63**

Here, we choose the DWM method [6] to generate the reference image, as

$$X^{ref} = \mathrm{DWM}(X) \quad (10)$$

where the X is the input image with noise, X^{ref} is the output image denoised by the DWM algorithm.

Then, the dictionary is trained by minimizing the following objective function,

$$\hat{D} = \arg\min_D \|D\alpha_{i,j} - R_{i,j}X^{ref}\|_2^2 + \sum_{i,j}\|\alpha_{i,j}\|_0 \quad (11)$$

The above minimization problem can be also solved by the coordinate relaxation method. First, for a fixed dictionary D, the problem turns into the sparse coding problem which can be solved by many greed algorithms, for example, the Orthogonal Matching Pursuit (OMP) algorithm. Then, after the coefficient α is calculated, D is updated by minimizing the function $f(D) = \|D\alpha_{i,j} - R_{i,j}X^{ref}\|_2^2$, which is a convex least square problem and can be solved by many methods, such as the MOD [18], and K-SVD algorithms [19].

III. Simulation Results

In this section, experiments have been carried out, and various grayscale images with different texture and features have been tested to verify the noise removal capability of the proposed algorithm. Besides, the proposed method is compared with several state of the art random-valued impulse noise removal methods, examples including the Luo-iterative method [5], directional weighted median (DWM) filter [6], contrast enhancement-based filter (CEF) [7], Trilateral filter [20], and the standard deviation for obtaining the optimal direction method (SDOOD) [21].

To evaluate denoised results quantitatively , two mathematical measurements, i.e., the peak signal to noise ratio (PSNR) and structural similarity (SSIM) are adopted. Readers can refer to [22] for a detailed understanding of the SSIM. For

TABLE II: Comparison of restoration results in SSIM for images corrupted by random-valued impulse noise

Method		Luo's [5]	CEF [7]	DWM [6]	SDOOD [21]	SBF [9]	Proposed
lena	30%	0.9303	0.9392	0.9384	0.9021	0.9187	**0.9571**
	40%	0.8807	0.9162	0.9280	0.8822	0.8817	**0.9453**
	50%	0.7957	0.8850	0.8905	0.8404	0.8119	**0.9189**
barbara	30%	0.7406	0.7467	0.7608	0.6939	0.6565	**0.8167**
	40%	0.6973	0.7018	0.7041	0.6554	0.6345	**0.7686**
	50%	0.6505	0.6581	0.6262	0.6181	0.5937	**0.6870**
house	30%	0.9473	0.9526	0.9616	0.9410	0.9563	**0.9759**
	40%	0.9095	0.9380	0.9537	0.9188	0.9199	**0.9561**
	50%	0.8359	0.9209	0.9450	0.8873	0.8594	**0.9467**

simplicity, suppose the X is the original image with size of $K{\times}L$, and $\hat{X}$ is the restoration version of X. Then, the PSNR value of $\hat{X}$ is defined as,

$$PSNR{=}10\log_{10}\frac{255^2}{\frac{1}{KL}\sum_{i=1}^{K}\sum_{j=1}^{L}\left[\hat{x}_{i,j}-x_{i,j}\right]^2} \tag{12}$$

and SSIM is used to measure the structural similarity of the restored image and original image,

$$SSIM{=}\frac{(2u_x u_{\hat{x}}+c_1)\,(2\sigma_{x\hat{x}}+c_2)}{(u_x^2+u_{\hat{x}}^2+c_1)(\sigma_x^2+\sigma_{\hat{x}}^2+c_2)} \tag{13}$$

where u_x and $u_{\hat{x}}$ denote the mean values of the original image and the restoration image, respectively; σ_x^2 and $\sigma_{\hat{x}}^2$ represent the variance of the original image and denoised image, respectively; $\sigma_{x\hat{x}}$ means the covariance between the original image and restoration result image, and c_1 and c_2 are two small variables used to avoid the unstable results.

In generally, the larger PSNR and SSIM values are, the better quality of the restored image is.

Before simulations, there are several parameters should be pre-defined: two constants λ_1 and λ_2 in model (5), and the parameter c in Eq. (7), and the maximum iteration number L_{max}. In our experiments, λ_1 is set as 0.1, λ_2 is empirically chosen as 1, $\lambda_3 = 1$, we assign $c = 25$ according to [12], and L_{max} is determined by experimental method, and by our extension experiments, $L_{max} = 2$ can achieve satisfying results. It is worth to note that these parameter values may are not the optimal values, and different applications require different optimal values.

In our simulations, various tested images were first contaminated by impulse noise with different noise ratios. Then, the corrupted images were processed by the proposed and all the compared methods, respectively, and the denoised images were measured by PSNR and SSIM. Due to space limitations, here we just show several denoised results. The others' compared methods are implemented by the optimal parameters and iteration numbers as their papers suggested for a fair comparison. The PSNR values are shown in Table I and the SSIM values are shown in Table II.

Fig. 1: 'lena' image with 40% random-valued impulse noise restored by different methods. (a)-(b) are the original and corrupted images; (c)-(h) are the restored images by different methods. (c) Luo-iterative method; (d) DWM; (e) CEF; (f) SDOOD; (g) SBF; (h) proposed method. Please zoom into pdf file for a detailed view.

In both Table I and II, the best values are marked in bold for easy comparison. From these two tables, one can find that, the restoration results generated by our proposed method get higher scores than those of other methods. This demonstrates that our method perform better than other methods in removing impulse noise. Especially, the proposed method has much improvement compared with the DWM though they two use the same noise detector, which further indicates that our sparse land based method has advantage in removing noise while preserving image details.

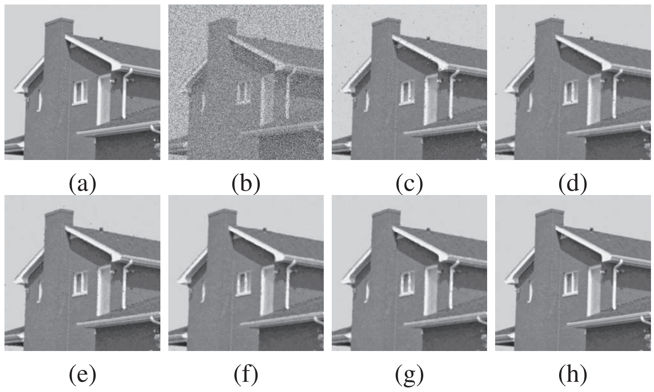

Fig. 2: 'house' image with 50% random-valued impulse noise restored by different methods. (a)-(b) are the original and corrupted images; (c)-(h) are the restored images by different methods. (c) Luo-iterative method; (d) DWM; (e) CEF; (f) SDOOD; (g) SBF; (h) proposed method. Please zoom into pdf file for a detailed view.

In order to subjectively assess the denoising performance of these compared methods, the restoration results of the standard test images 'lena' and 'house' are shown in Fig. 1 and Fig. 2. In Fig. 1, the original 'lena' image is corrupted by random-valued impulse noise with 40% noise ratio, and restored by all the tested methods. In Fig 2, the 'house' image is corrupted by 50% random-valued impulse noise, and processed by the compared algorithms. For a better visual view, we enlarged

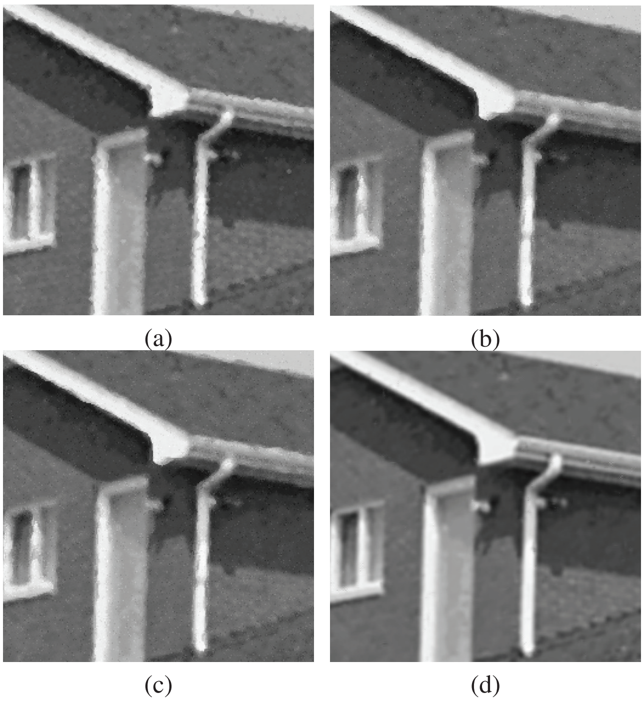

Fig. 3: Enlarged parts of several restoration results in Fig. 2. (a) CEF; (d) SD-OOD; (e) SBF; (f) proposed method. Please zoom into pdf file for a detailed view.

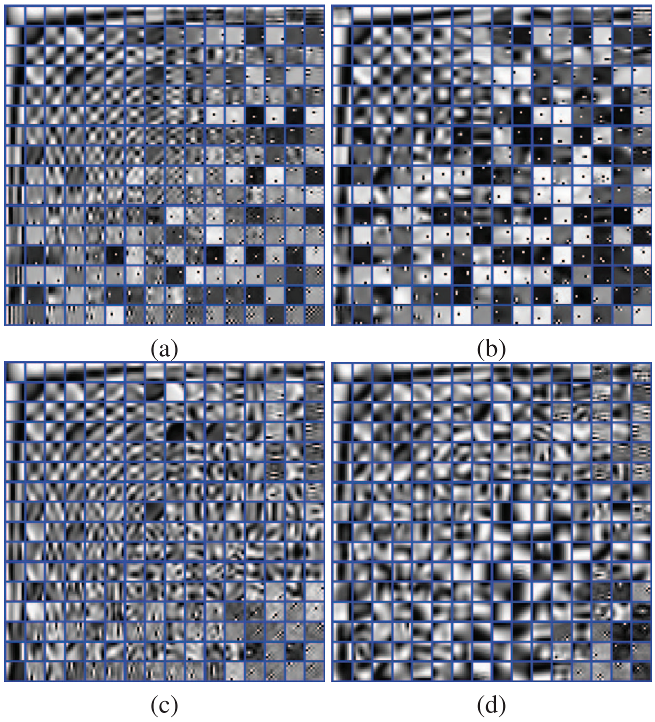

Fig. 4: Trained dictionaries for 'lena' and 'house' images with 40% random-valued impulse noise: (a) and (c) dictionaries trained for 'lena' image; (b) and (d) dictionaries trained for 'house' image. (a) and (b) dictionaries trained directly from noisy images; (c) and (d) dictionaries trained by our method.

the restored images in Fig. 2, and shown portion of them in Fig 3 to exhibit the detailed information such as edges and textures. From these three figures, it can be seen that most of other methods not only cannot suppress the noise well but also damage image edges and bring many artifacts. Though some techniques such as CEF and SDOOD can preserve most image details, they fail to remove the noise thoroughly, especially when the noise density is high (e.g. 50% impulse noise, please refer to Fig. 3 for comparison). By contrast, the restored images generated by our method present high image quality with few impulse noise, which demonstrates that the proposed method has the capability of perfectly removing impulse noise while preserving most of the image details.

Fig. 4 shows the dictionaries trained for 'lena' and 'house' images contaminated by 40% random-valued impulse noise. Above row are dictionaries trained directly from the noise images, bottom are the dictionaries trained by our proposed method from the reference images. As can be seen, the directly trained dictionaries contain many noisy atoms which are not suitable for reconstructing the original image, while the trained dictionaries by the proposed method contain few noise, and keep most of the textures and details information of the original images, hence they are more appropriate for representing the objective images.

IV. Conclusion

This paper presents a novel sparse representation model with fuzzy weights for removing impulse noise. The fuzzy weight matrix can be generated by any noise detectors. It is used to exclude the impulse noise from the minimization energy function, which helps to force the reconstructed image more close to the original image. Moreover, the noisy image serviced as global prior is also introduced into the proposed sparse land model. This makes the proposed model more robust to the noise detectors. Besides, the dictionary used in our method is trained from some reference images which keeps the similar texture information with the original image, therefore, is more suitable for representing the noisy image. Simulation results demonstrate that the proposed method outperforms several state of the art impulse noise removing methods.

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